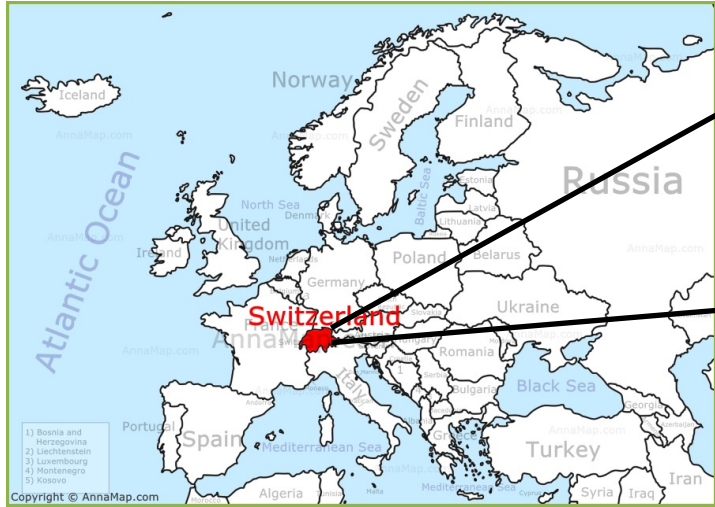


# Simulations of stellarator boundary turbulence in the space of magnetic geometries

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# Where are we, behind the screen

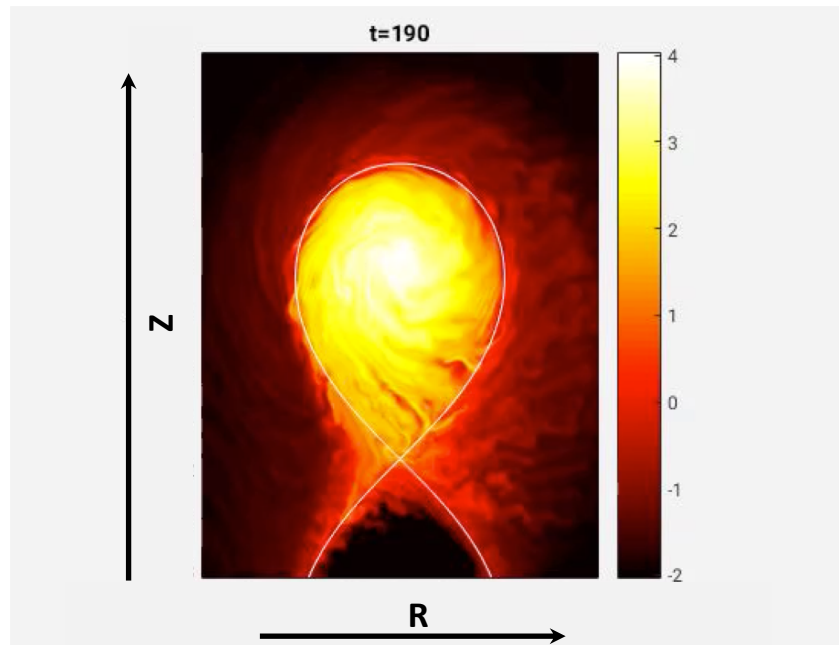


As tokamaks, stellarators need to address the issue of how to best exhaust heat and particles.

Goals concerning power exhaust:

- ❖ exhaust power without damaging materials  
→ radiate and spread the heat on target
- ❖ maintain core performance  
→ control impurity dilution and ionization/radiation fronts
- ❖ allow easy pumping of neutrals  
→ maximize neutral pressure close to target

Important player in this game is **turbulence**!



GBS simulation of plasma turbulence in a tokamak with a single null

Braginskii [Reviews of Plasma Physics, 1965] derived, starting from kinetic theory, a set of fluid equations that is asymptotically valid in the limit of high plasma collisionality ( $\nu^* \gg 1$ ) and thus adequate in the 'cold boundary'.

continuity 
$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{V}_\alpha) = 0$$

momentum 
$$m_\alpha n_\alpha \frac{d_\alpha \mathbf{V}_\alpha}{dt} = -\nabla p_\alpha - \nabla \cdot \underline{\underline{\pi_\alpha}} + e_\alpha n_\alpha (\mathbf{E} + \mathbf{V}_\alpha \times \mathbf{B}) + \mathbf{R}_\alpha$$

energy 
$$\frac{3}{2} n_\alpha \frac{d_\alpha T_\alpha}{dt} = -p_\alpha \nabla \cdot \mathbf{V}_\alpha - \nabla \cdot \mathbf{q}_\alpha + Q_\alpha^{visc} + Q_\alpha$$

... + Maxwell equations

*~ viscosity* (pointing to  $\underline{\underline{\pi_\alpha}}$ )

*~ resistivity* (pointing to  $\mathbf{R}_\alpha$ )

*~ heat conductivity* (pointing to  $\mathbf{q}_\alpha$ )

Braginskii equations describe the plasma dynamics on time scales ranging from  $\Omega_{ce}^{-1} \sim 10^{-11} \text{ s}$  up to  $\tau_E \sim 1 \text{ s}$ .



Zeiler [IPP report 5/88, 1999] derived, starting from Braginskii equations, a reduced set of equations valid in the limit of “low-frequency” ( $\omega \ll \Omega_{ci}$ ) and “large scale” turbulence ( $(k_{\perp} \rho_s)^2 \ll 1$ ) thus adequate in the boundary.

For example, in the cold ion ( $T_i = 0$ ) electrostatic limit ( $\partial_t B = 0$ ):

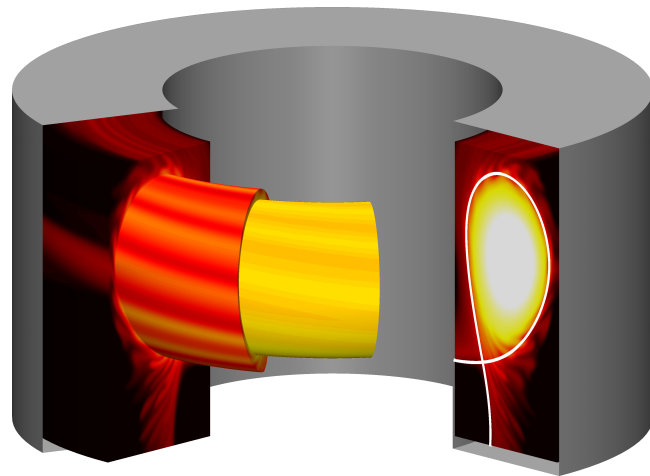
electron continuity	$\frac{\partial n}{\partial t} = -\nabla \cdot [n(\mathbf{V}_E + \mathbf{V}_{de} + V_{\parallel e} \mathbf{b})] ,$	$\sim$ drive for curvature-driven modes
charge conservation	$\nabla \cdot \left( \frac{en}{B\omega_{ci}} \frac{d_{i0}}{dt} \nabla_{\perp} \phi \right) = \nabla_{\parallel} j_{\parallel} - \nabla \cdot (en \mathbf{V}_{de}) ,$	$\sim$ destabilizes drift-waves
electron momentum	$m_e n \frac{d_{e0} V_{\parallel e}}{dt} = -\nabla_{\parallel} p_e + en \nabla_{\parallel} \phi - 0.71 n_e \nabla_{\parallel} T_e + en_e \nu_{\parallel} j_{\parallel} - \frac{2}{3} \nabla_{\parallel} G_e$	
ion momentum	$m_i n \frac{d_{i0} V_{\parallel i}}{dt} = -\nabla_{\parallel} p_e ,$	
electron energy	$\frac{3}{2} n_e \frac{d_e T_e}{dt} = -p_e \nabla \cdot \mathbf{V}_e + 0.71 \frac{T_e}{e} \nabla_{\parallel} j_{\parallel} + \chi_{\parallel e} \nabla_{\parallel}^2 T_e + \nabla \cdot \left( \frac{5 n T_e}{2 e B} \mathbf{b} \times \nabla T_e \right)$	

The Global Braginskii Solver (GBS) [Giacomin, JCP 2022] developed over the last  $\sim 15$  years:

- solves **Zeiler's equations** in a toroidal domain of rectangular cross-section,
- given an equilibrium  $\mathbf{B}$ , 2D or 3D, with arbitrary magnetic topology [Coelho et al, NF 2022],
- given density and temperature **sources**,
- with **sheath** boundary conditions [Loizu et al, PoP 2012],
- with coupling to a kinetic neutral model [Mancini et al, NF 2024].

**Quasi-steady state = balance between source, turbulence, sheath losses**

Cross-validation of turbulence codes (GBS, GRILLIX, TOKAM3X) with experiments on TCV has been carried out [Oliveira et al, NF 2022].



**GBS simulation of TCV shot #65402 @1s**

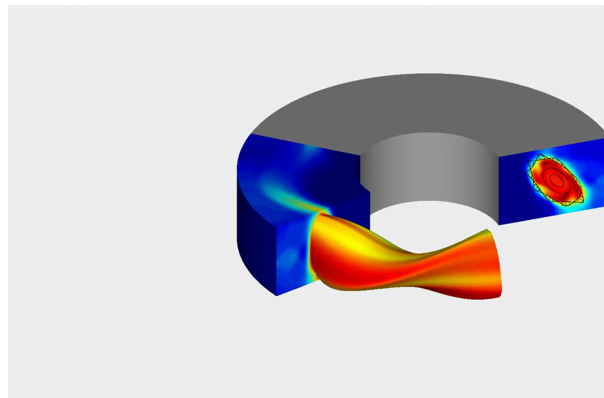
[M. Giacomin, PhD thesis 2022]

GBS numerical scheme includes:

- Explicit time-advance using Runge-Kutta fourth-order scheme,
- Spatial derivatives evaluated with fourth-order finite difference scheme,
- Arakawa scheme for the Poisson brackets ( $\mathbf{ExB}$  advection),
- Density and velocity grids are staggered in two directions,
- MPI parallelization in  $(\mathbf{x}, \mathbf{y}, \mathbf{z})$  with  $\mathbf{z}$  the 'toroidal direction'.

Typical stellarator simulation on JFRS-1:

- Grid size  $(n_x, n_y, n_z) \sim (200)^3$ ,
- 1 node per few  $(\mathbf{x}, \mathbf{y})$  planes, total of  $\sim 40$  nodes,
- Simulation time  $\sim 50'000$  node-hours.

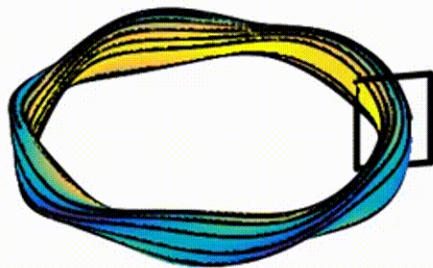


[Coelho et al, NF 2022]

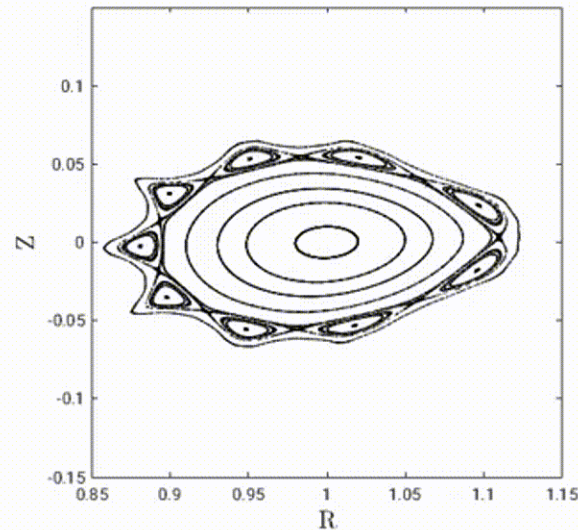
A stellarator vacuum field can be described by a potential satisfying Laplace's equation.

$$\nabla \times \mathbf{B} = 0 \quad \mathbf{B} = \nabla V$$

$$\nabla \cdot \mathbf{B} = 0 \quad \nabla^2 V = 0$$



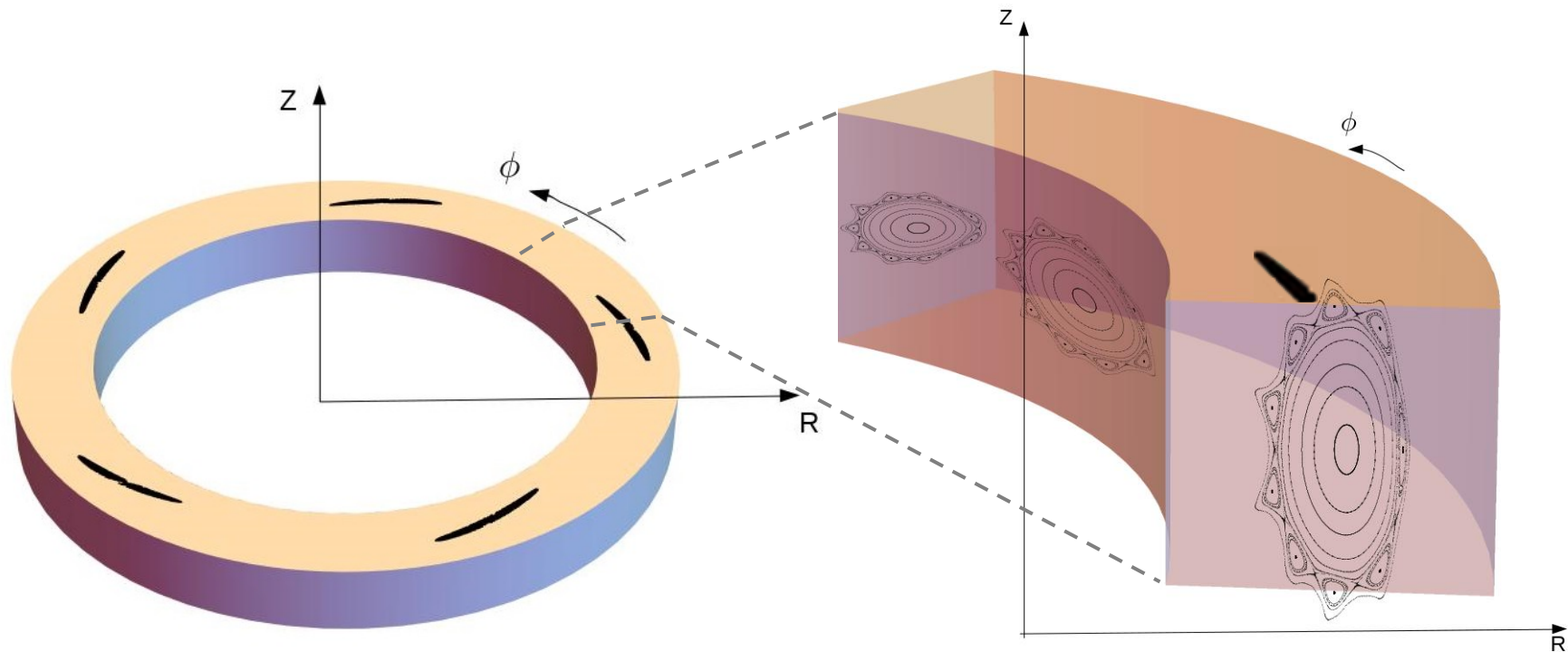
$$V(R, \phi, Z) = \phi + \sum_{m,l} V_{m,l}(R, \phi, Z)$$



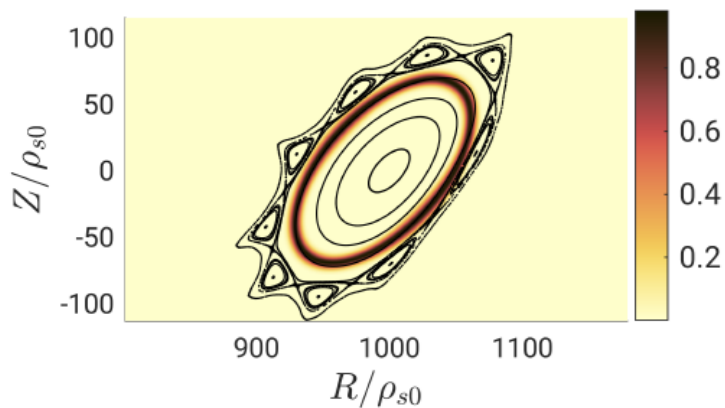
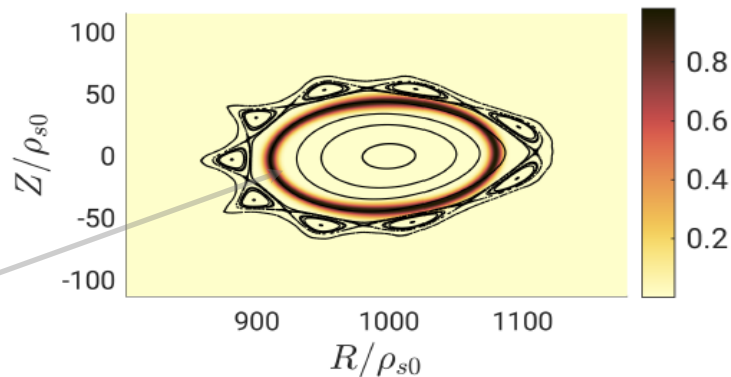
low shear,  
 $\iota = 0.5-0.55$

Dommasck potentials [Dommaschk, CPC 1986] form complete basis for the vacuum solution in a torus.

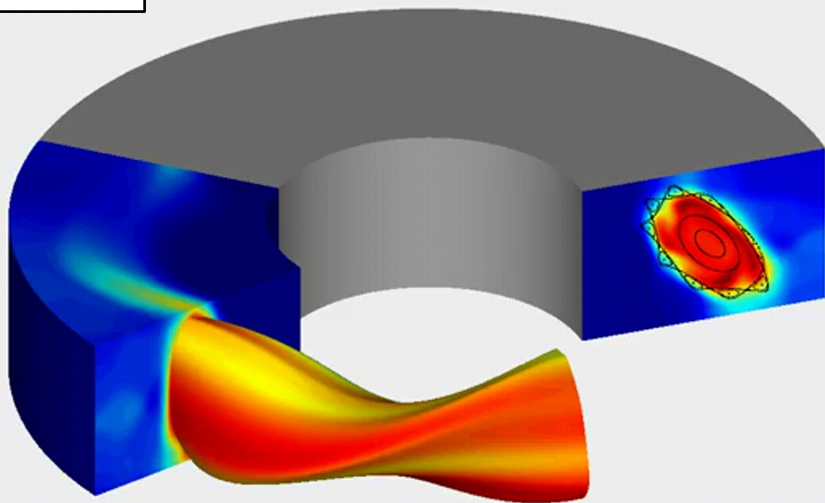
We shape the wall so that islands intersect top/bottom



$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{\Gamma}_{\text{ExB}} + \nabla \cdot \mathbf{\Gamma}_{\text{dia}} + \nabla \cdot \mathbf{\Gamma}_{\parallel e} = \mathcal{S}_n$$



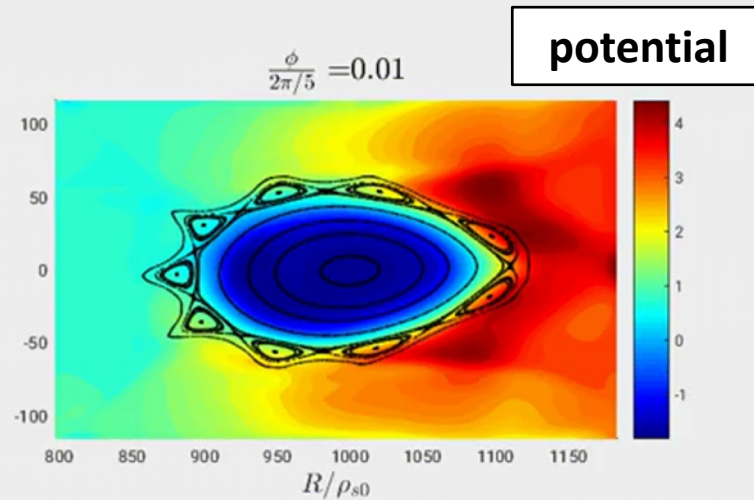
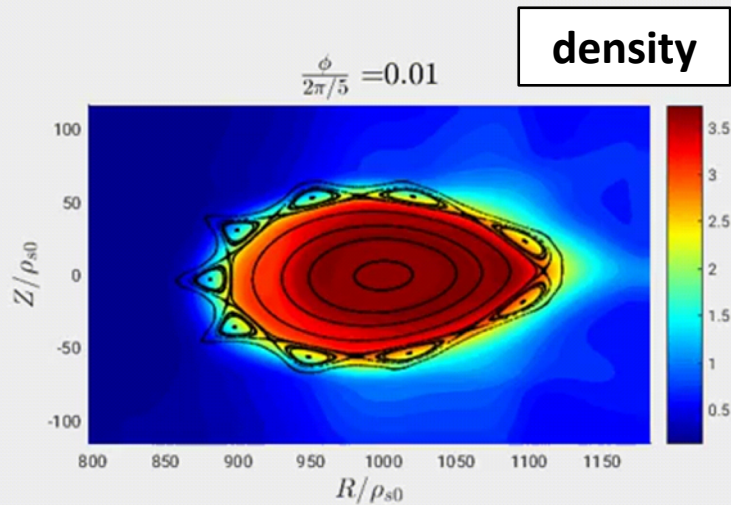
density

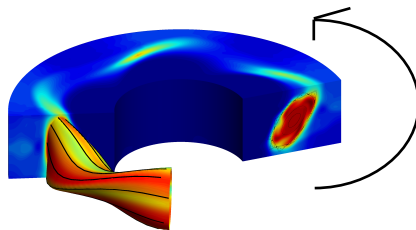
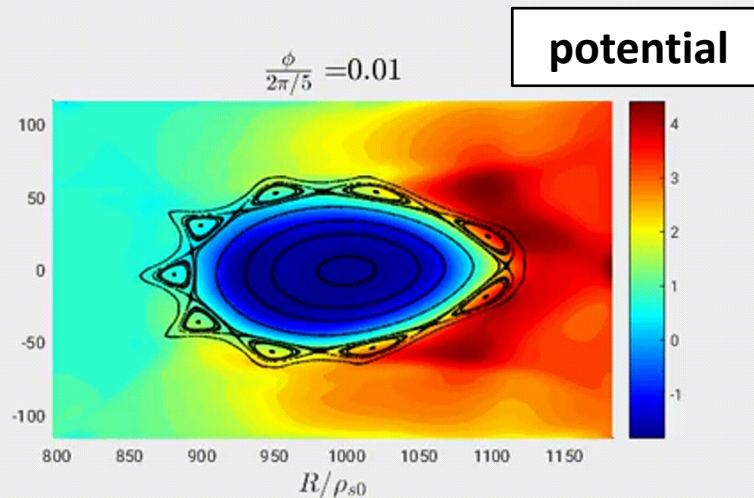
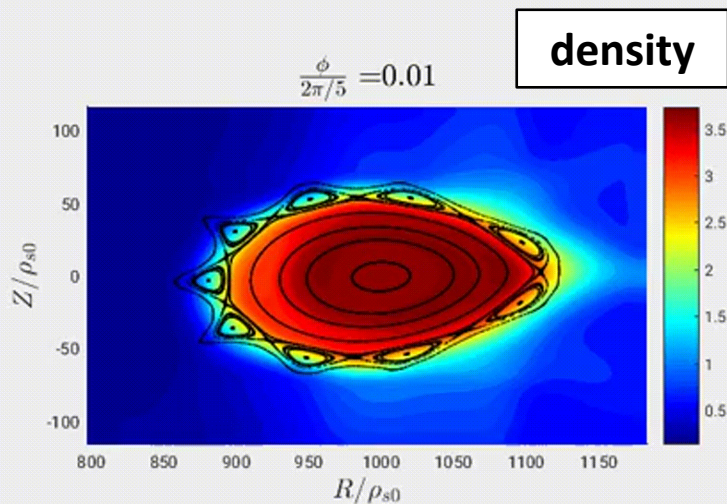


equilibrium  $\rightarrow \langle f \rangle_t$

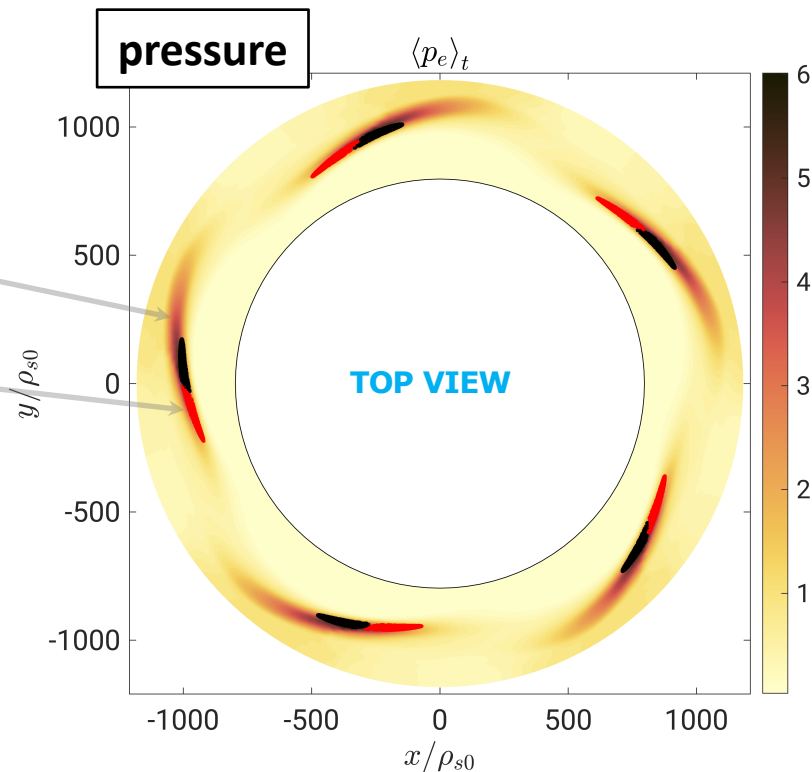
fluctuations  $\rightarrow \tilde{f} = f - \langle f \rangle_t$

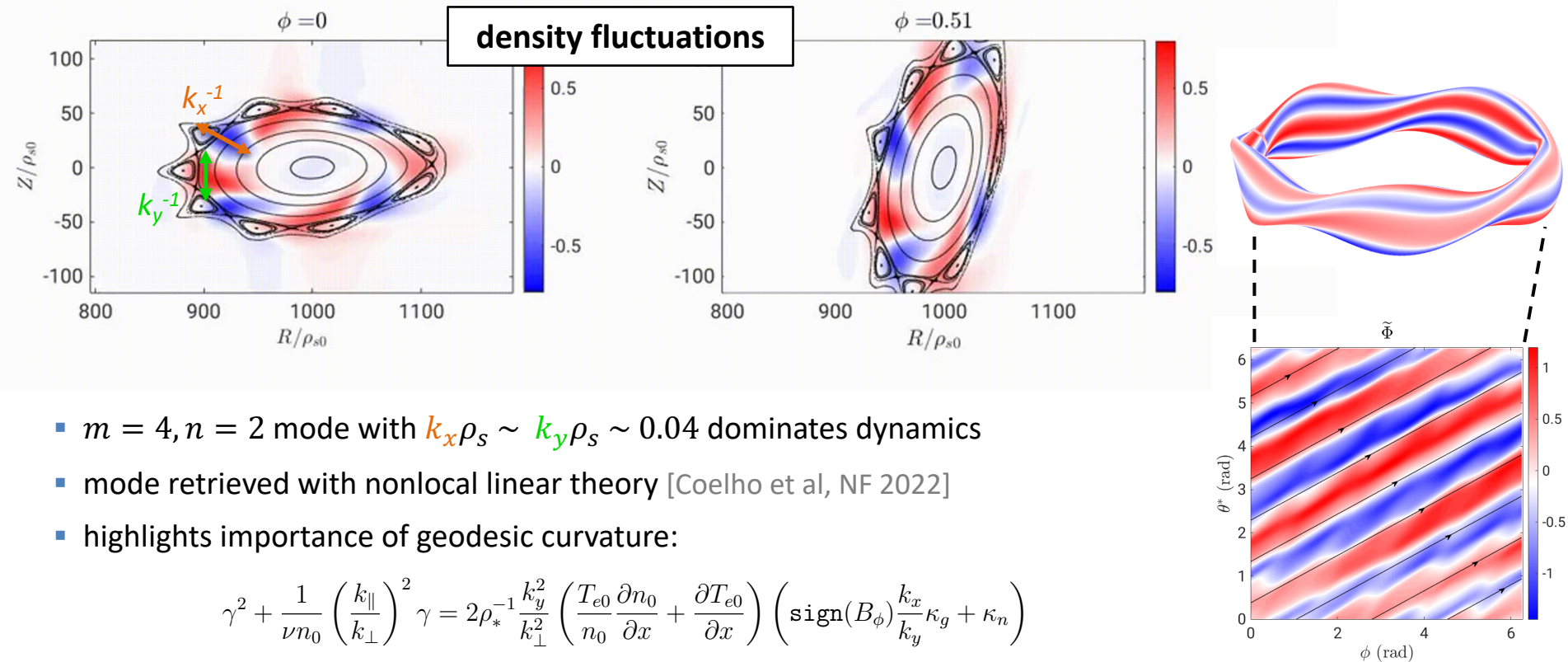






- The heat deposition pattern on the targets is  
as expected from the footprints of the islands.



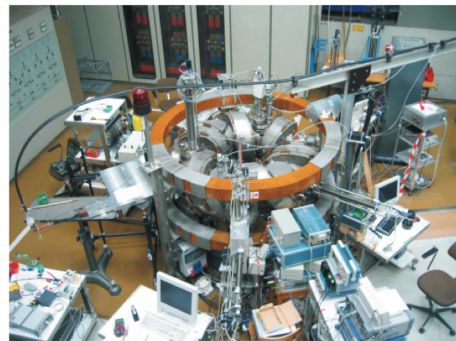


- $m = 4, n = 2$  mode with  $k_x \rho_s \sim k_y \rho_s \sim 0.04$  dominates dynamics
- mode retrieved with nonlocal linear theory [Coelho et al, NF 2022]
- highlights importance of geodesic curvature:

$$\gamma^2 + \frac{1}{\nu n_0} \left( \frac{k_{\parallel}}{k_{\perp}} \right)^2 \gamma = 2\rho_*^{-1} \frac{k_y^2}{k_{\perp}^2} \left( \frac{T_{e0}}{n_0} \frac{\partial n_0}{\partial x} + \frac{\partial T_{e0}}{\partial x} \right) \left( \text{sign}(B_{\phi}) \frac{k_x}{k_y} \kappa_g + \kappa_n \right)$$

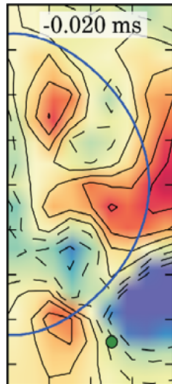
- mode breaks the discrete symmetry of the stellarator! [Coelho et al, submitted to NF]

[Coelho et al, PPCF 2023]

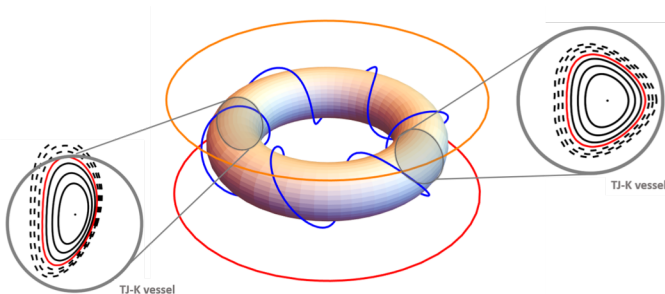
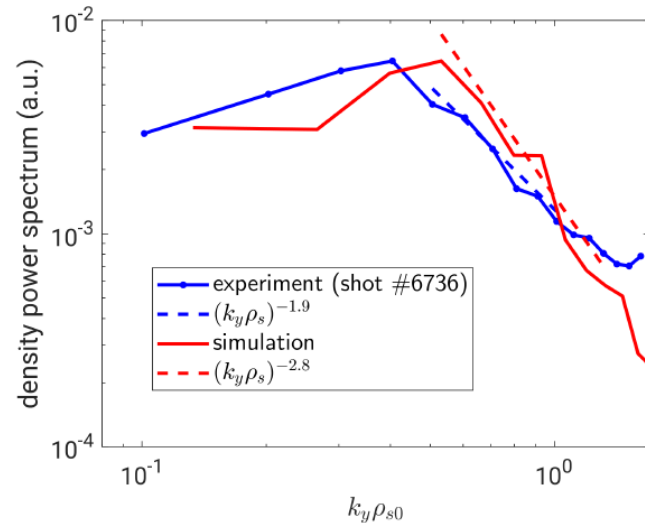
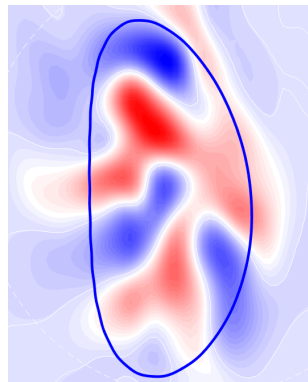


TJ-K stellarator (6-field period)

density fluct. - experiment    density fluct. - simulation



[Fuchert et al, PPCF 2013]



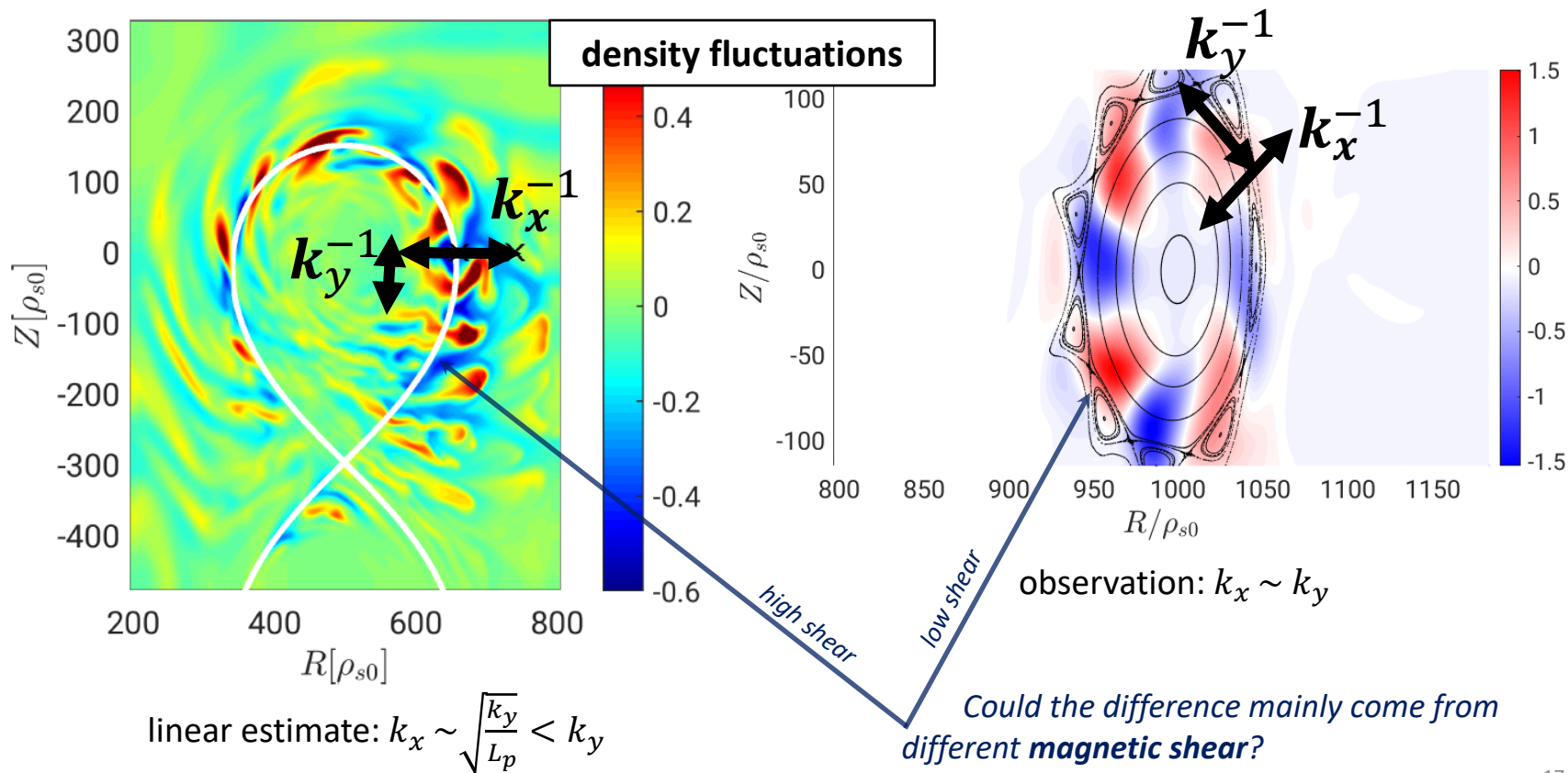
Used GBS code to simulate plasma dynamics in TJ-K with real sources.

Model valid in whole device (except no neutral physics included).

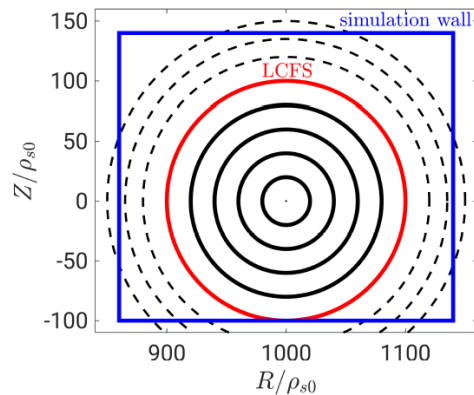
Reproduced the fluctuations spectrum, dominant  $m=4, n=1$  mode.



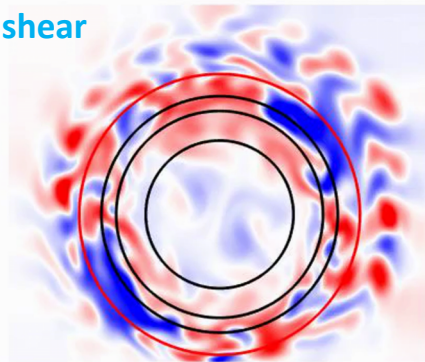
# Differences between tokamak/stellarator simulations might be explained by magnetic shear



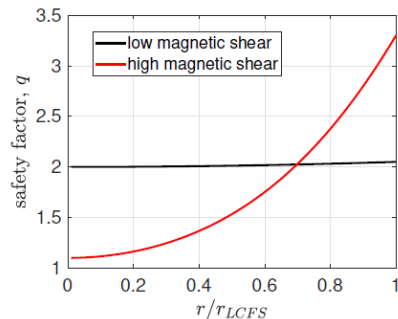
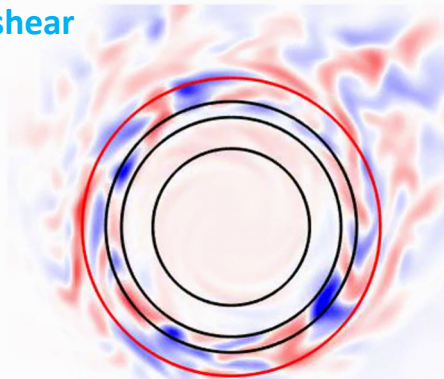
[Tecchiolli et al, in preparation]



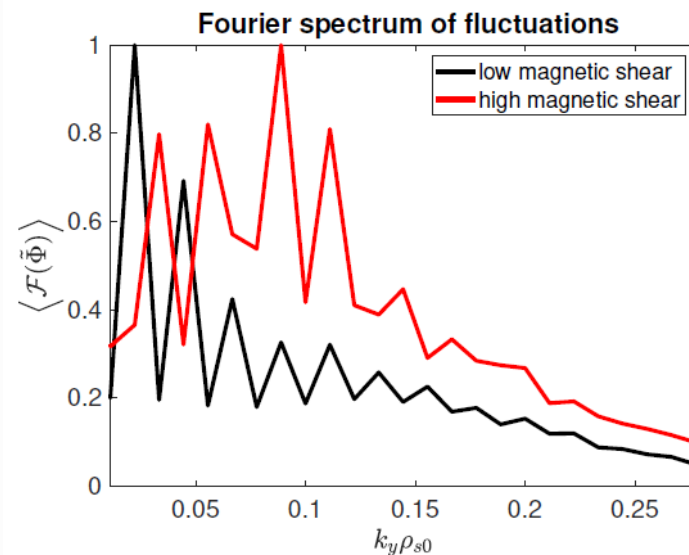
low shear



high shear

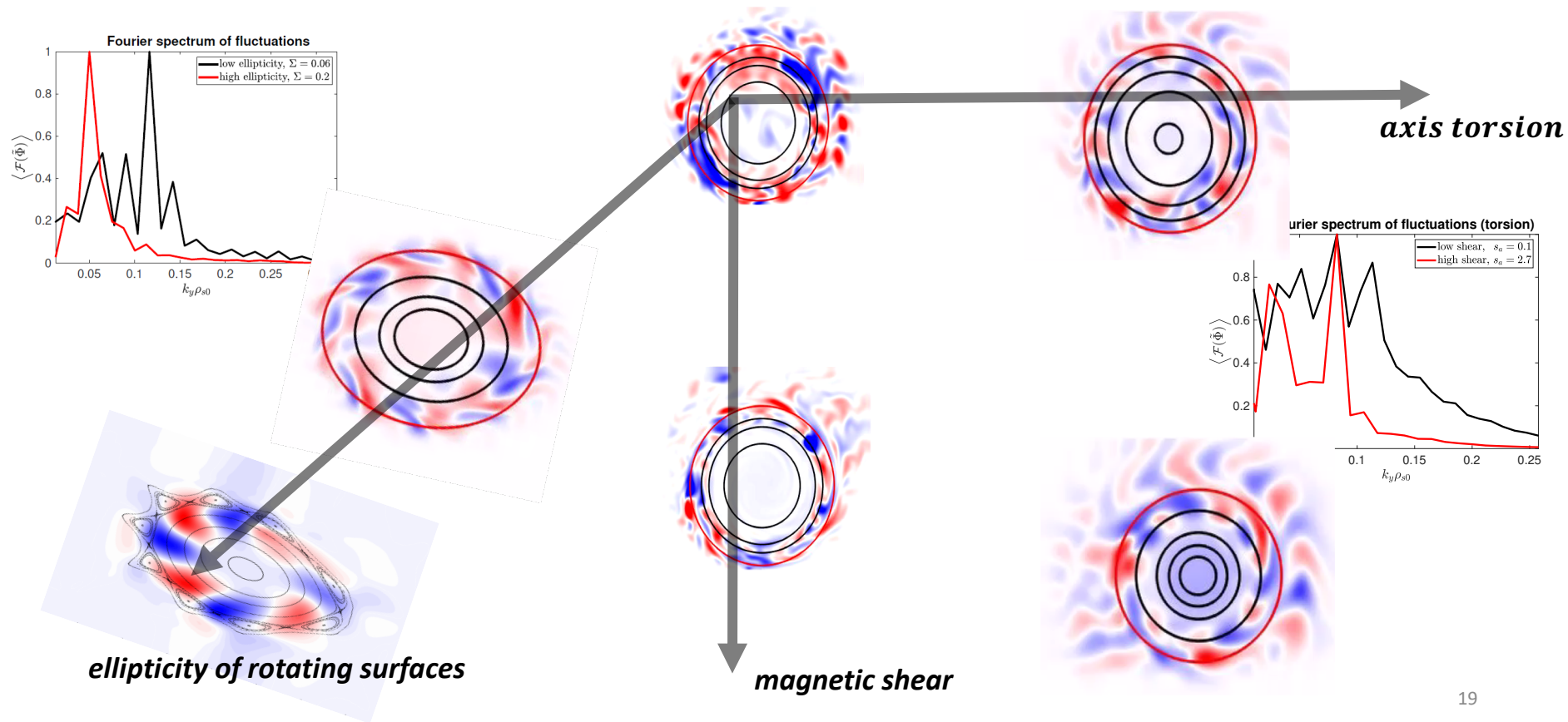


two circular tokamak  
configurations

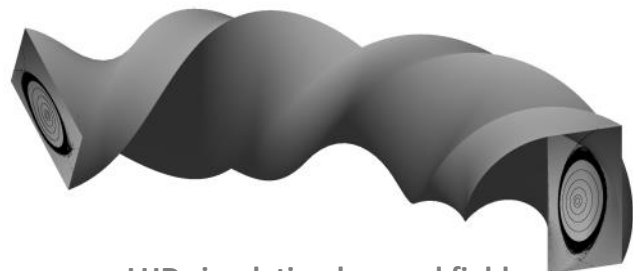




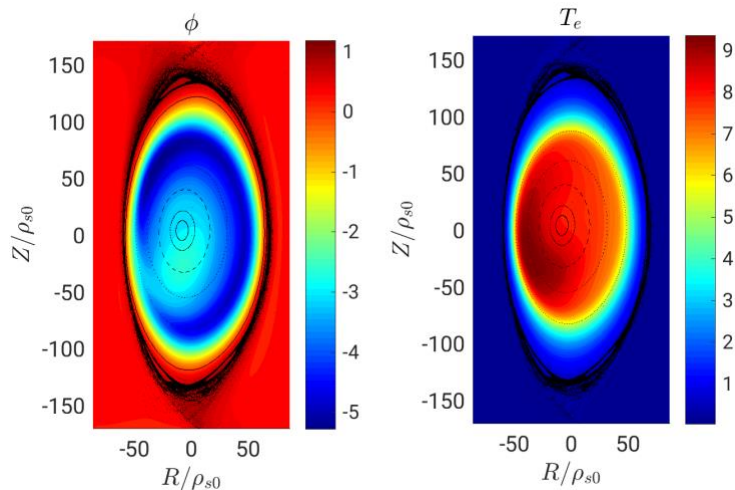
# We explored the effect of ellipticity and torsion



- ❖ LHD has high-shear, high ellipticity, small torsion: what is the nature of boundary turbulence?
- ❖ We might be able to reproduce soft density limits as observed experimentally.
- ❖ Including the **neutral physics** in the simulations might allow studying detachment.



LHD simulation box and field



- ❖ GBS is the first code to carry out a global simulation of boundary fluid turbulence in a stellarator.
- ❖ Low- $m$  coherent modes that break stellarator periodicity tend to develop, at least in low-shear.
- ❖ We have reproduced the fluctuation spectrum in the TJ-K stellarator experiment.
- ❖ We have explored the effect of global shear, axis-torsion, and near-axis-ellipticity on turbulence.
- ❖ We are currently simulating realistic large-scale configurations such as LHD, but also W7-AS.